

## Fifth Semester Examination – 2007 Automata Theory

**Full Marks – 70**

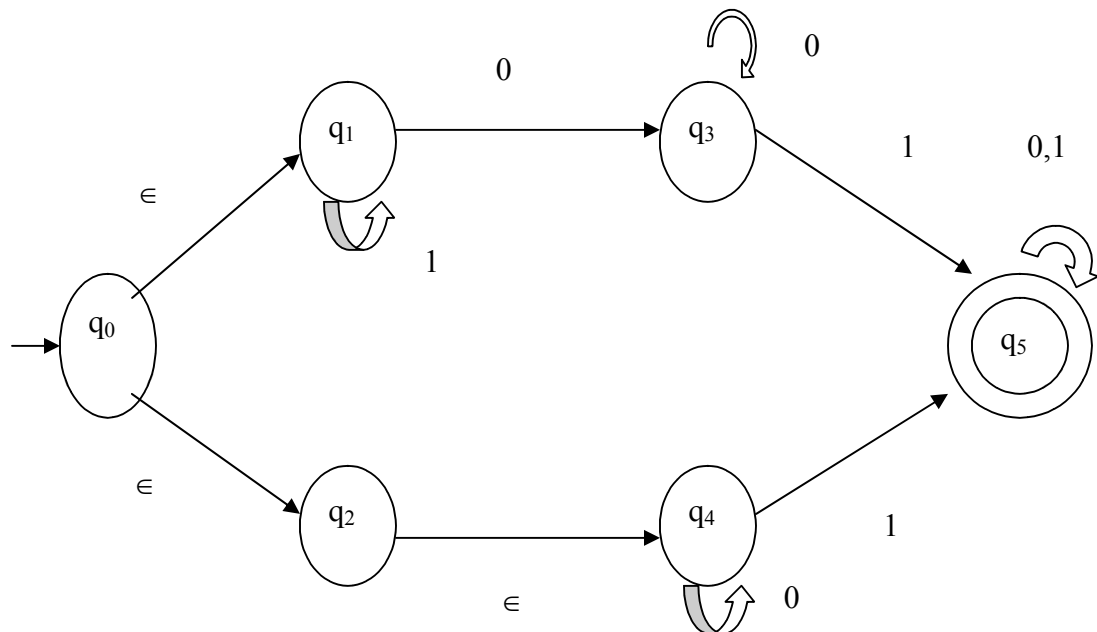
**Time – 3 hours**

Answer Question No. 1 which is compulsory and any five from the rest.

The figures in the right-hand margin indicate marks.

1. Answer the following questions: ( 2 x 10

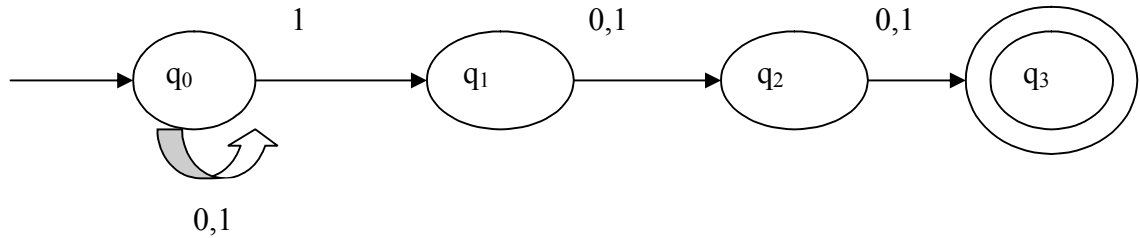
- (a) Given an alphabet  $\Sigma$ , what do you mean by a language  $L$  over  $\Sigma$  ?
- (b) Devise a DFA which accepts all strings ending with 1. (Assume that  $\Sigma = \{0,1\}$  )
- (c) Define a NFA.
- (d) You have a DFA  $M_1$  which accepts a language  $L_1$  and another DFA  $M_2$  which accepts the language  $L_2$ . Devise a  $\epsilon$ -NFA which accepts the language  $L_1 \cup L_2$
- (e) Find the  $\epsilon$ -closure of the state  $\{q_0\}$  from the following diagram:



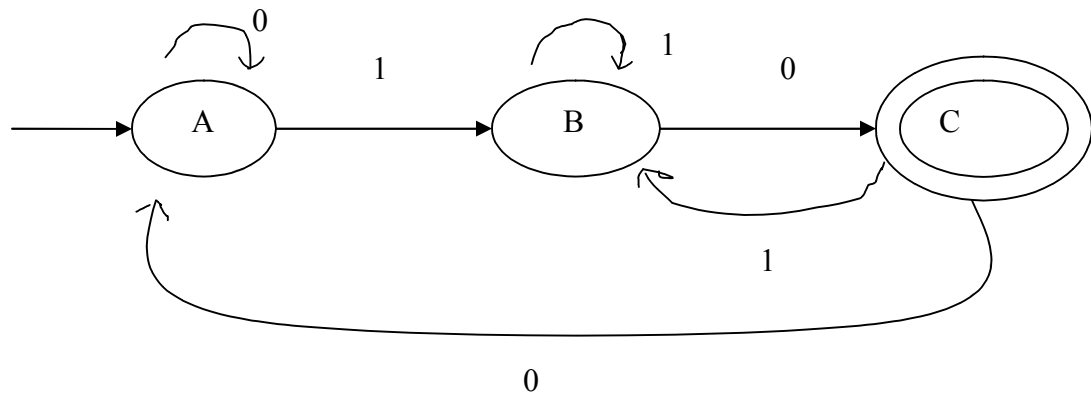
- (f) Define a pushdown automation.
- (g) Find the language generated by the following context-free grammar:  

$$S \rightarrow a S a \mid b S b \mid a \mid b$$
- (h) When is a context-free grammar said to be ambiguous?
- (i) When is a language  $L$  said to be Turing recognizable? Turing decidable?

- (j) Let  $f, g, h, k : \mathbf{N} \rightarrow \mathbf{N}$ . Show that if  $f = O(h)$  and  $g = O(k)$ , then  $fg = O(hk)$ .
- 2.
- (a) Devise a DFA which accepts all binary strings which are divisible by 4. (5)
- (b) Convert the following NFA to its equivalent DFA. (5)

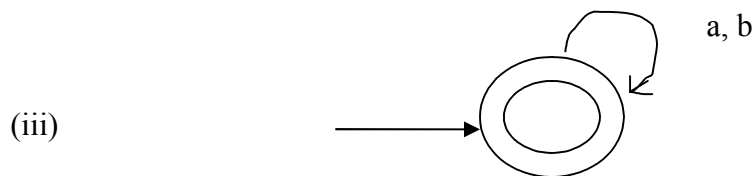
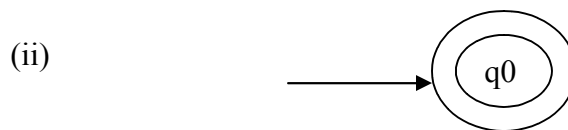
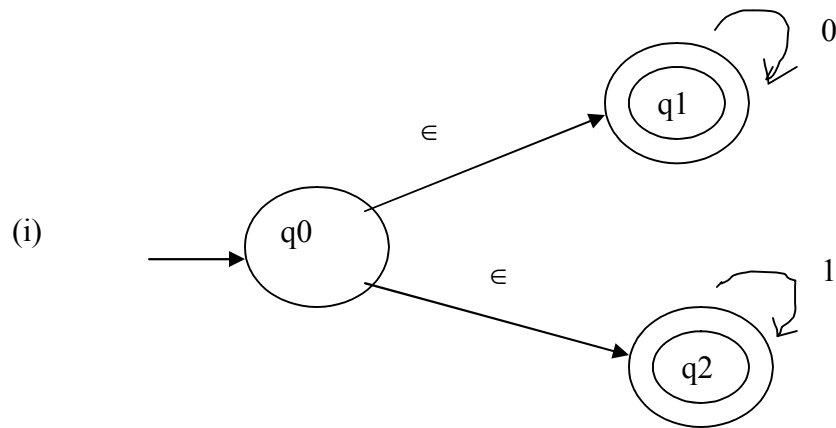


- 3.
- (a) From the diagram of a DFA given below, devise a regular grammar which generates the language of the DFA. (4)



- (b) Describe the regular expression corresponding to the language generated by the DFA in 3(a) above. (3)

(c) Find the languages generated by the following DFA's:



4.

(a) State and prove the pumping lemma for a regular language. (5)

(b) Use the pumping lemma to show that the language (5)

$$L = \{0^n 1^n : n \geq 1\}$$

is not regular.

5.

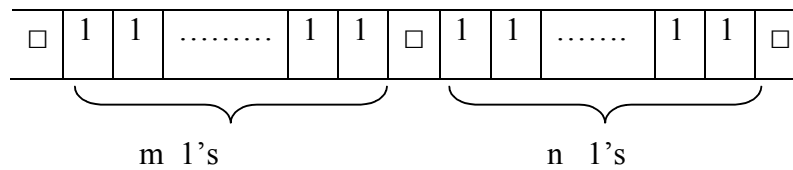
(a) State and prove the pumping lemma for context-free languages. (5)

(b) Show that the language  $L = \{a^n c b^n : n \geq 1\}$  is accepted by a PDA. (5)

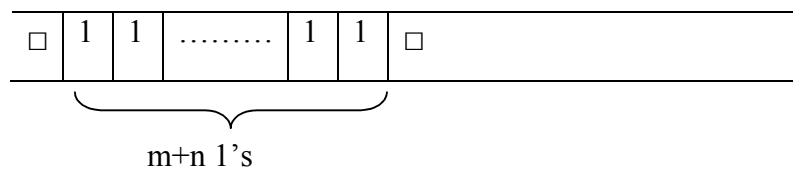
6.

- (a) Define a Turing machine. (5)
- (b) Devise a Turing machine which on a given pair of positive integers(m,n) as input, will produce m+n as output. [Assume that the positive integers are represented by a unary alphabet{1}]. (5)

Therefore the input on the tape is:



The output should be



□ = blank symbol

7.

- (a) Show that the set of all Turing recognizable languages is countable. (5)
- (b) Show that if a language L is decidable so is its complement. (5)

8.

- (a) When a function  $f : \Sigma^* \rightarrow \Sigma^*$  said to be computable? (Where  $\Sigma$  is a given alphabet). (2)
- (b) When is a language A said to be mapping reducible to another language B ( both over the same alphabet)? (2)
- (c) Define the class NP. Describe three problems belonging to the class NP. (6)

---X---